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KINETIC EQUATIONS OF HIGH-INTENSITY HEAT AND MASS TRANSFER

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A system of basic kinetic equations of high-intensity heat and mass transfer in capillary-porous bodies is obtained by averaging the equations of heat and mass transfer with variable coefficients.

The use of averaged relations between quantities obtained on the basis of the equations of heat and mass transfer is a useful method of generalizing experimental data and developing engineering methods for the calculation of heat- and mass-transfer processes. This "integral approach" was adopted in [1] to find the dependence of the heat flow on the rate of drying and heating of a body, and was further developed in [2] and elsewhere. It might be expected that an analogous approach would be just as useful in more general and more complex problems.

In the present work, this approach is extended to the case of high-intensity heat and mass transfer, in which filtrational mass transfer begins to play a significant role. This allows kinetic equations to be obtained for the heat flow $j_q(\tau)$ and the mass flow of material leaving the body — the total flow $j_m(\tau)$ and its filtrational (molar) component $j_p(\tau)$. The problem is solved without any assumptions as to the constancy of the coefficients of heat and mass transfer or the kinds of contact between the material and the surrounding medium.

Intensive heat and mass transfer in a capillary-porous body is described by a system of nonlinear equations [1, 3]:

$$\frac{\partial T}{\partial \tau} = \nabla (a_q \nabla T) + \frac{\varepsilon r}{c_q} \frac{du}{d\tau}, \tag{1}$$

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$$\frac{\partial u}{\partial \tau} = \nabla \left(a_m \nabla u + a_m \delta \nabla T + \frac{k_P}{\gamma_0} \nabla P \right), \tag{2}$$

$$\frac{\partial P}{\partial \tau} = \nabla (a_P \nabla P) - \frac{\varepsilon}{c_W} \frac{\partial u}{\partial \tau}. \tag{3}$$

The total pressure of the vapor—gas mixture inside the body, producing the filtrational flow of a mass of associated material, is made up of the pressure of the vapor forming as the liquid evaporates and the external pressure, including the hydrostatic pressure of the absorbed liquid.

The boundary conditions at the surface of the body are written in a form that is appropriate for any means of supply of heat and removal of mass of the material leaving the body:

$$\lambda_q (\nabla T)^* = j_q^* (\tau), \tag{4}$$

$$a_m \gamma_0 (\nabla u)^* + a_m \delta \gamma_0 (\nabla T)^* + k_P (\nabla P)^* = -j_m^* (\tau), \tag{5}$$

$$c_W a_P \gamma_0 (\nabla P)^* = -j_P^* (\tau), \tag{6}$$

where $(\nabla T)^*$, $(\nabla u)^*$, and $(\nabla P)^*$ are the local gradients of temperature, mass content, and total pressure at the surface of the body.

In Eqs. (1)-(3), we change to values averaged over the volume of the body and at the same time transform the integrals over the volume using Gauss's theorem:

$$\frac{d\bar{T}}{d\tau} = \frac{1}{V} \int_F a_q \nabla T dF + \frac{\bar{\varepsilon} \bar{r}}{\bar{c}_q} \frac{d\bar{u}}{d\tau}, \tag{7}$$

$$\frac{d\bar{u}}{d\tau} = \frac{1}{V} \int_F \left[a_m \nabla u + a_m \delta \nabla T + \frac{k_P}{\gamma_0} \nabla P \right] dF, \tag{8}$$

$$\frac{d\bar{P}}{d\tau} = \frac{1}{V} \int_F a_P \nabla P dF - \frac{\bar{\varepsilon}}{\bar{c}_W} \frac{d\bar{u}}{d\tau}. \tag{9}$$

When the heat and mass flow is averaged over the surface of the body, the boundary conditions in Eqs. (4)-(6) take the form

$$\frac{1}{F} \int_F \lambda_q (\nabla T)^* dF = \frac{1}{F} \int_F j_q^* (\tau) dF = j_q(\tau), \tag{10}$$

$$\frac{1}{F} \int_F \gamma_0 \left[a_m (\nabla u)^* + a_m \delta (\nabla T)^* + \frac{k_P}{\gamma_0} (\nabla P)^* \right] dF$$

$$= -\frac{1}{F} \int_F j_m^* (\tau) dF = -j_m (\tau), \tag{11}$$

$$\frac{1}{F} \int_F c_W a_P \gamma_0 (\nabla P)^* dF = -\frac{1}{F} \int_F j_P^* (\tau) dF = -j_P (\tau). \tag{12}$$

Taking account of Eqs. (10)-(12), Eqs. (7)-(9) may now be rewritten to give

$$\frac{d\bar{T}}{d\tau} = \frac{1}{R_V \gamma_0 c_q^s} j_q (\tau) - \frac{\bar{\varepsilon} \bar{r}}{R_V \gamma_0 \bar{c}_q} j_m (\tau), \tag{13}$$

$$\frac{d\bar{u}}{d\tau} = -\frac{1}{R_V \gamma_0} j_m (\tau), \tag{14}$$

$$\frac{d\bar{P}}{d\tau} = -\frac{1}{R_V \gamma_0 c_W^s} j_P (\tau) + \frac{\bar{\varepsilon}}{R_V \gamma_0 \bar{c}_W} j_m (\tau), \tag{15}$$

where $R_V = V/F$; c_q^s and c_W^s are values of the specific heat of the material and the vapor—gas mixture averaged over the surface of the body at the given instant.

Only the basic thermodynamic parameters of heat and mass transfer appear in Eqs. (13)-(15). It is simple to obtain from them kinetic equations for the integral flow of heat and mass. To this end, we divide Eq. (13) by Eq. (14):

$$\frac{d\bar{T}}{d\bar{u}} = \frac{\bar{\varepsilon}\bar{r}}{\bar{c}_q} - \frac{1}{c_q}\frac{j_q(\tau)}{j_m(\tau)},$$

and hence obtain the heat flow

$$j_q(\tau) = \frac{c_q^s}{\bar{c}_q}\bar{r}j_m(\tau)\left(\bar{\varepsilon} - \frac{\bar{c}_q}{\bar{r}}\frac{d\bar{T}}{d\bar{u}}\right).$$

The quantity $(\bar{c}_q/\bar{r})(d\bar{T}/d\bar{u})$ is the Rebinder number Rb. Using this notation, the expression for the heat flow takes the final form

$$j_q(\tau) = \frac{c_q^s}{\bar{c}_q}\bar{r}j_m(\tau)(\bar{\varepsilon} - \mathrm{Rb}). \tag{16}$$

As is evident from Eq. (14), the mass flow of the material leaving the body surface is determined by the integral rate of drying

$$j_m(\tau) = -R_V\gamma_0\frac{d\bar{u}}{d\tau}. \tag{17}$$

To determine the filtrational component of the total mass flow, we divide Eq. (15) by Eq. (14):

$$\frac{d\bar{P}}{d\bar{u}} = \frac{1}{c_W^s}\frac{j_P(\tau)}{j_m(\tau)} - \frac{\bar{\varepsilon}}{\bar{c}_s},$$

and hence obtain

$$j_P(\tau) = \frac{c_W^s}{\bar{c}_W}j_m(\tau)\left(\bar{\varepsilon} + \bar{c}_W\frac{d\bar{P}}{d\bar{u}}\right). \tag{18}$$

The dimensionless quantity $\bar{c}_W(d\bar{P}/d\bar{u})$ characterizes the change in the total pressure of the vapor—gas mixture of liquid inside the body when the mass content of the body changes by unity over an infinitesimally small interval of time; it is proportional to the degree of filling of the pores by liquid at a given temperature. This is a fundamental quantity in the kinetics of filtrational mass transfer.

We shall call $c_W(dP/du)$ the Mikhailov number Mi,

$$\mathrm{Mi} = c_W\frac{dP}{du}. \tag{19}$$

Thus, taking Eq. (19) into account, the equation for the filtrational mass flow may be rewritten in a form similar to that for the heat flow:

$$j_P(\tau) = \frac{c_W^s}{\bar{c}_W}j_m(\tau)(\bar{\varepsilon} + \mathrm{Mi}). \tag{20}$$

Thus, Eqs. (16), (17), and (20) determine the kinetics of the mean-integral values over the surface of the body of the flow of heat and of mass; they constitute a system of basic kinetic equations for high-intensity internal heat and mass transfer.

Lykov's basic equation of drying kinetics [1] follows from Eq. (16). In the course of drying, liquid is lost from the body by evaporation from the surface ($\varepsilon = 1$). If the coefficients of heat and mass transfer are assumed constant, $\bar{c}_q = c_q^s$. In addition, in the drying process, $d\bar{u}/d\tau < 0$ and $\mathrm{Rb} < 0$. On these assumptions, Eq. (16) is transformed to Lykov's equation

$$j_q(\tau) = R_V\gamma_0 r\frac{d\bar{u}}{d\tau}(1 + \mathrm{Rb}).$$

The density of the heat flow supplied to the material in the course of intensive drying may also be determined using Bulygin's number Bu. To demonstrate this, we divide Eq. (15) by Eq. (13):

$$\frac{d\bar{P}}{d\bar{T}} = \frac{\dfrac{\bar{\varepsilon}}{\bar{c}_W}j_m(\tau) - \dfrac{1}{c_W^s}j_P(\tau)}{\dfrac{1}{c_q^s}j_q(\tau) - \dfrac{\bar{\varepsilon}\bar{r}}{\bar{c}_q}j_m(\tau)},$$

and hence obtain

$$\frac{\bar{r}\bar{c}_W}{\bar{c}_q}\frac{d\bar{P}}{d\bar{T}}=\frac{\bar{\varepsilon}\bar{r}\,j_m(\tau)-\frac{\bar{c}_W}{c_W^s}\bar{r}j_P(\tau)}{\frac{\bar{c}_q}{c_q^s}j_q(\tau)-\bar{\varepsilon}\bar{r}\,j_m(\tau)},$$

where $\bar{r}(\bar{c}_W/\bar{c}_q)(d\bar{P}/d\bar{T}) = \mathrm{Bu}$.

The heat flow can now be written in terms of Bu:

$$j_q(\tau)=\frac{c_q^s}{\bar{c}_q}\bar{\varepsilon}\bar{r}\,j_m(\tau)\left(1+\frac{1}{\mathrm{Bu}}\right)-\frac{c_q^s\bar{c}_W\bar{r}}{\bar{c}_q c_W^s \mathrm{Bu}}\,j_P(\tau). \tag{21}$$

Using Eq. (21), which is a linear combination of the basic kinetic equations in Eqs. (16) and (20), the relation between Bu and the basic numbers Rb and Mi of the kinetics of heat and mass transfer may be established. Taking into account Eq. (20), we transform Eq. (21) to give

$$j_q(\tau)=\frac{c_q^s}{\bar{c}_q}\bar{r}j_m(\tau)\left(\varepsilon-\frac{\mathrm{Mi}}{\mathrm{Bu}}\right). \tag{22}$$

Comparing Eqs. (22) and (16) leads to the required relation

$$\mathrm{Bu}=\mathrm{Mi}/\mathrm{Rb}.$$

Thus, by means of the basic kinetic equations of internal heat and mass transfer obtained above, the heat flow and the mass flow of associated material at any instant after the beginning of the process may be determined using only a very limited number of parameters — the integral drying rate, the latent heat of vaporization, and the numbers ε, Rb, and Mi. These parameters can be determined from a knowledge simply of the kinetics of the temperature, mass-content, and total-pressure fields (hydrobarothermal fields) in the material, together with its pore structure.

This method of calculation eliminates the need for experimental determinations of numerous heat- and mass-transfer characteristics of the material and of the coefficients of external heat and mass transfer.

The basic kinetic equations given above may be used for the calculation of high-intensity processes such as mechanical drying, high-temperature drying, the firing of ceramic and other articles, etc.

NOTATION

T, absolute temperature; u, mass content; P, total pressure; ε, phase-conversion factor; r, latent heat of vaporization of liquid; c_q, specific heat of wet material; $c_W = \mu \Pi b/\gamma_0 RT$, specific heat of water vapor; λ_q, thermal conductivity coefficient of material; a_q, a_m, a_P, coefficients of heat diffusion, mass diffusion, and convective mass diffusion, respectively; δ, coefficient of thermal-gradient heat transfer; k_P, permeability of material to air; Π, porosity of material; γ_0, density of absolutely dry material; μ, mass of 1 mole of absorbed liquid; b, degree of filling of pore by liquid; R, universal gas constant; $j_q(\tau)$, $j_m(\tau)$, $j_P(\tau)$, densities of heat flow, total mass flow, and filtrational component of mass flow; Rb, Rebinder number; Mi, Mikhailov number; Bu, Bulygin number; V, volume of material; F, surface of material; R_V, hydraulic radius of body; τ, time. Indices: s, value averaged over the surface of the body; *, local value at the surface of the body. A bar over a quantity denotes the value averaged over the volume of the body.

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